VELOCITY PROFILE OF THE TURBULENT FLOW OF WEAK POLYMER SOLUTIONS IN A TUBE

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Experimental data relating to the velocity profiles of the turbulent flow of polyox, guar resin, and polyacrylamide solutions (10^{-6} to 6.10^{-4} g/cm^3) in a 35-mm diameter tube are presented. Analytical relationships obtained by expressing these profiles in the form of a three-layer model enable us to relate the form of the velocity profiles to the resistance coefficient and the Reynolds number of the flow.

In order to discover the laws of turbulent flow in weak polymer solutions exhibiting the Toms effect [1] we require to study the structures of these flows. The velocity profile is the most accessible characteristic of the turblent flow of solutions in tubes for experimental determination.

The results of earlier measurements of velocity profiles close to the tube wall in a laminary sublayer [2, 3] revealed no appreciable difference between the shapes of the velocity profiles of water and weak polymer solutions. Any special influence of the polymer additives on the shape of the profile might be expected to appear outside this zone.

The velocity profiles extending across the tube were measured by means of microtubes of the total-head type. This method was reasonably simple and enabled a large number of measurements to be made. The investigations were carried out in a hydraulic apparatus of the closed type in a tube 35 mm in diameter, a description of which was presented in [4]. As polymer samples we used polyox, guar resin and polyacrylamide of various concentrations.

Certain difficulties associated with the degradation of the polymer solutions and with the phenomenon known in the literature as the "tube defect" [5-7] were eliminated (as far as possible) by only recording the results of those experiments in which the degradation of the solution led to an increase in resistance of no more than 5% above the initial value, and a difference of no more than 1-2% between the rates of flow calculated from the measured profiles and recorded in the MIR-1 flowmeter. The flow data determined in these two ways are presented in Table 1, from which we see that in the experiments under consideration no tube defect appeared. The effect of the degradation of the solution on the shape of the velocity

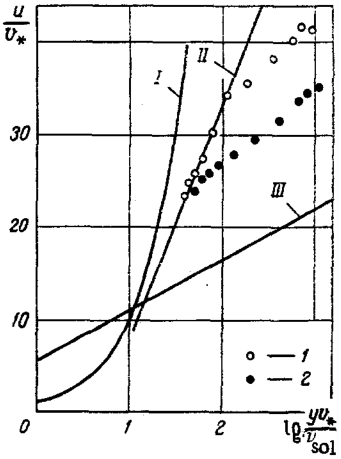

Fig. 1. Velocity profiles of turbulent flows in a tube 35 mm in diameter involving polyox solutions of concentration $c = 10^{-5}$ g/cm^3: 1) fresh solution (S = 65%); 2) degraded solution (S = 53%); I) by the equation $u/v_* = y(v_*/\nu_{sol})$; II) (1); III) (2).

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TABLE 1. Velocity Profile of the Flow of a Solution of Guar Resin of Concentration $c = 3.10^{-4}$ g/cm^3 in a Tube with a Diameter of $d = 35$ mm

y, mm	r, mm	v, m/sec	$\Delta r^2(v_i + v_{i+1})$ mm^2 · m/sec	u/v_*	$\lg(yv_*/\nu_{sol})$
17,50	0	7,05	18,6	38,3	3,33
16,35	1,15	7,05	46,0	38,3	3,31
15,35	2,15	6,97	73,9	37,8	3,28
14,35	3,15	6,97	101,0	37,8	3,25
13,35	4,15	6,88	128,0	37,4	3,22
12,35	5,15	6,90	154,1	37,5	3,19
11,35	6,15	6,81	180,0	37,0	3,16
10,35	7,15	6,78	208,0	36,8	3,11
9,35	8,15	6,72	233,0	36,5	3,07
8,35	9,15	6,65	231,5	36,1	3,02
7,35	10,15	6,56	295,6	35,6	2,96
6,35	11,15	6,45	300,0	35,0	2,90
5,35	12,15	6,34	153,0	34,4	2,82
4,85	12,65	6,30	162,8	34,2	2,78
4,35	13,15	6,22	161,3	33,8	2,73
3,85	13,65	6,19	172,2	33,6	2,68
3,35	14,15	6,12	182,7	33,2	2,62
2,85	14,65	6,05	174,0	32,8	2,55
2,35	15,15	5,94	106,0	32,2	2,47
2,05	15,45	5,85	98,6	31,8	2,41
1,75	15,75	5,75	142,3	31,2	2,34
1,35	16,15	5,62	150,3	30,6	2,23
1,05	16,45	5,51	107,9	30,0	2,12
0,75	16,75	5,28	103,4	28,8	1,97
0,55	16,95	5,06	59,5	27,5	1,84
0,45	17,05	4,85	32,7	26,3	1,75
0,35	17,15	4,50	26,2	24,5	1,64
0,25	17,25	4,21	40,8	22,8	1,59
			$\sum_i = 3843{,}3$ (mm^2 · m /sec)		

Note: $q \cong 6$ liters/sec; $V_S = 6.2$ m/sec (by instrument); $\lambda = 0.00704$; $\nu_{sol} = 1.48 \cdot 10^{-6}$ m^2/sec; $v_* = 0.184$ m/sec; $Re = 1.46 \cdot 10^5$; $V_S = (1/2R^2)\sum_i (v_i + v_{i+1}) \Delta r^2 = 6.26$ m/sec (by profile).

profile is shown in Fig. 1, which gives the velocity profiles of two flows of a polyox solution of concentration $c = 10^{-5}$ g/cm^3, namely a fresh solution, with a reduction in resistance of S = 65%, and the same solution in which this figure had changed to S = 53% as a result of degradation. We see from Fig. 1 that the velocity profiles of the two flows differ very considerably from one another. The difference in the absolute velocities close to the wall is ~11%, while in the center of the tube it is no greater than 1%. On the basis of this fact, in all subsequent experiments we measured from the wall of the tube in the direction of the axis of the flow.

The results of these investigations were then expressed in semilogarithmic coordinates $u/v_*-\lg(yv_*/\nu_{sol})$ (Fig. 2). On the basis of these experimental data we may reasonably assume that there are two limiting forms of velocity profile for flows of weak polymer solutions:

$$u/v_* = 26 \lg(yv_*/\nu_{sol}) - 18.2 \tag{1}$$

for concentrations greater than or equal to the limiting value and

$$u/v_* = 5.75 \lg(yv_*/\nu_{sol}) + 5.5, \tag{2}$$

when there is no reduction in the resistance, either because of the low polymer concentration or because of the low values of the dynamic velocity $v_* = \sqrt{\tau_{sol}/\rho} < v_{*\lim}$ [4]. The intermediate case of velocity distributions over the tube cross section for the flows of weak polymer solutions will be described by Eq. (1) for $y < y_1$, and by

$$u/v_* = 5.75 \lg(yv_*/\nu_{sol}) + B, \tag{3}$$

where $B > 5.5$, for $y > y_1$.

At the point with the coordinate y_1, both equations [(1) and (3)] are satisfied at the same time, enabling us to determine the free term B of Eq. (3) in the form

$$B(\gamma) = 20.25 \lg(y_1 v_*/\nu_{sol}) - 18.2. \tag{4}$$

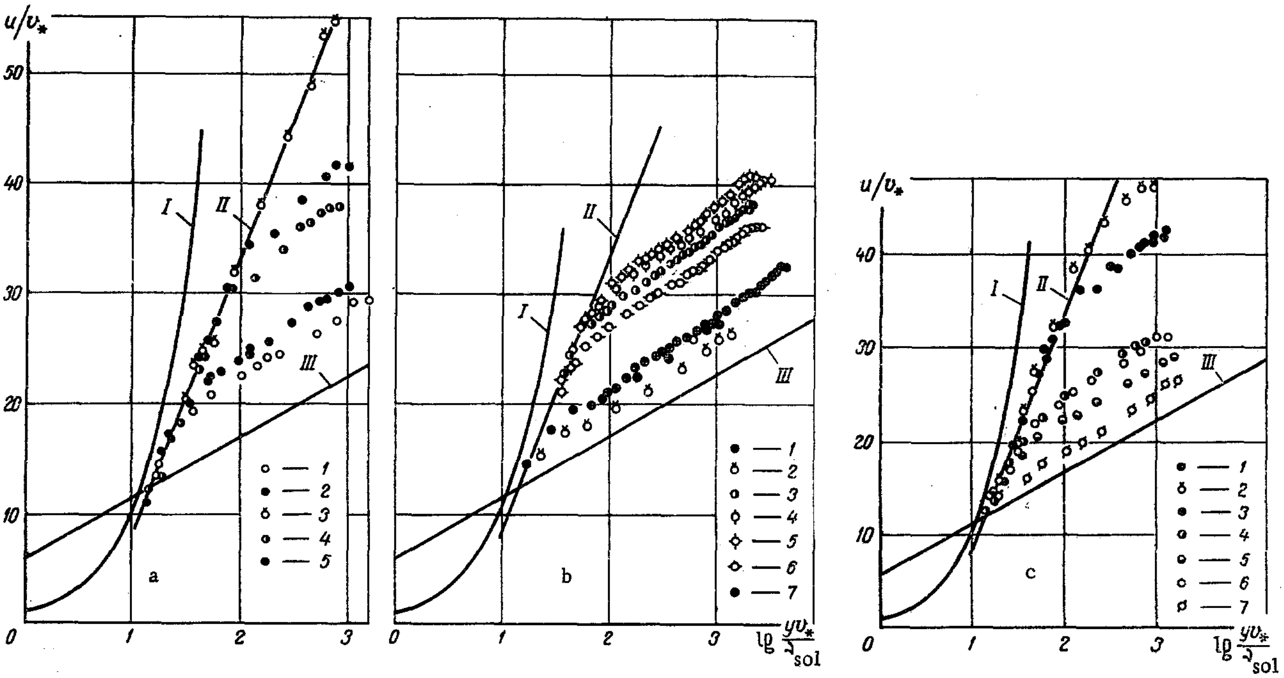

Fig. 2. Velocity profiles of turbulent flows in a 35-mm diameter tube involving the following polymer solutions: a) polyox: 1) c = $2\cdot10^{-6}$ g/cm^3, q = 2.6 liters/sec; 2) respectively $2\cdot10^{-6}$ and 1.6; 3) $2.5\cdot10^{-5}$ and 1.7; 4) 10^{-5} and 1.7; 5) 10^{-5} and 1.9; b) guar resin: 1) c = $3\cdot10^{-4}$ g/cm^3, q = 1.86 liters/sec; 2) $6\cdot10^{-5}$ and 1.82; 3) $3\cdot10^{-4}$ and 6.0; 4) $3\cdot10^{-4}$ and 7.5; 5) $6\cdot10^{-4}$ and 6.45; 6) $1.5\cdot10^{-4}$ and 6.0; 7) $6\cdot10^{-5}$ and 6.5; c) polyacrylamide: 1) c = $7\cdot10^{-5}$ g/cm^3, q = 4.17 liters/sec; 2) $1.4\cdot10^{-4}$ and 4.28; 3) $3.5\cdot10^{-5}$ and 2.6; 4) 3.5 $\cdot10^{-5}$ and 1.4; 5) $5\cdot10^{-6}$ and 2.5; 6) $1.4\cdot10^{-5}$ and 2.5; 7) 10^{-6} and 2.5; curves I, II, and III have the same meanings as in Fig. 1.

The value of the parameter $\gamma = y_1/R$ changes from $y_1/R = 1$, corresponding to the limiting form of velocity profile (1) to $y_1/R \cong 83/\mathrm{Re}\sqrt{\lambda}$ when the velocity profile coincides with that of the flow of water in the tube.

The shape of the profiles of polymer solutions flowing in tubes enables us to find the resistance coefficients corresponding to specified rates of flow of the liquid in the tube (or a specified Reynolds number). For this purpose we first have to find the relationship between the mean rate of flow and the maximum velocity in the cross section of the tube.

The mean flow rate of the liquid in the tube may be expressed thus:

$$v_S = \frac{1}{\pi R^2}\int\limits_{\Omega} u\,ds, \quad \text{where } ds = 2\pi(R-y)\,dy. \tag{5}$$

The velocity distribution over the tube cross section should be taken in the following form:

$$\begin{aligned} u/v_* &= 26\lg(yv_*/\nu_{sol}) - 18.2, \quad 0 \lesssim y < y_1 \\ u/v_* &= 5.75\lg(yv_*/\nu_{sol}) + 20.25\lg(y_1v_*/\nu_{sol}) - 18.2, \quad y_1 < y < R. \end{aligned} \tag{6}$$

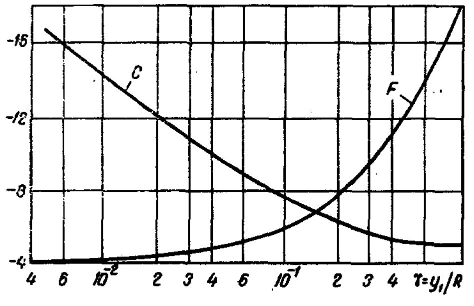

Fig. 3. Coefficients F and C as functions of the parameter $\gamma = y_1/R$.

Integration of Eq. (5) with due allowance for (6) gives the following relationship between the mean velocity and the maximum velocity on the axis of the tube:

$$v_S = U - v_*F(\gamma), \tag{7}$$

where

$$F(\gamma) = 1.09(3.75 + 17.6\gamma - 4.4\gamma^2), \quad \gamma = y_1/R. \tag{8}$$

In Eq. (8) we have introduced the factor 1.09 so as to make the calculation agree with Nikuradze's experimental data for the flow of water in a tube [10].

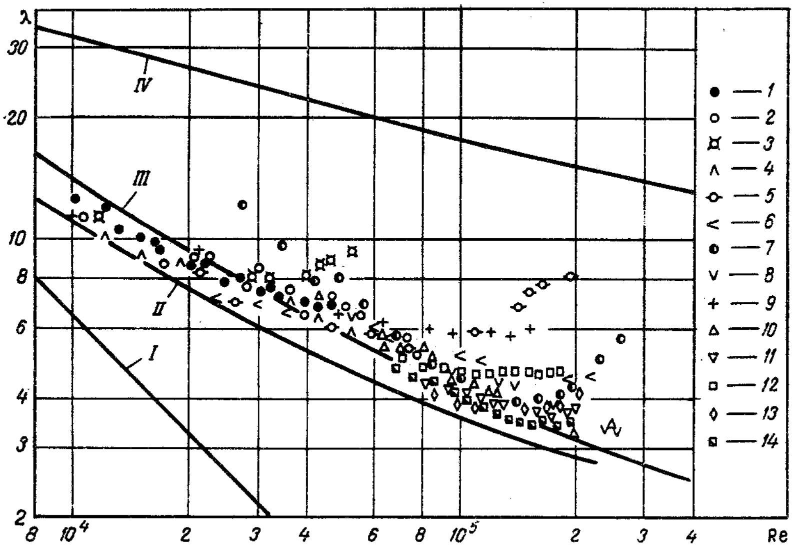

Fig. 4. Resistance coefficient $\lambda = 8\tau_{sol}/\rho v_s^2$ as a function of the Reynolds number $Re = v_s d/\nu_{sol}$ for high-efficiency solutions flowing in tubes: 1) d = 9.75 mm, c = 10^{-5} g/cm^3; 2) respectively 9.75 and $5 \cdot 10^{-5}$; 3) 9.75 and $5 \cdot 10^{-6}$; 4) 9.75 and 10^{-4}; 5) 20.9 and 10^{-5}; 6) 20.9 and $5 \cdot 10^{-5}$; 7) 35.5 and 10^{-5}; 8) 35.5 and $5 \cdot 10^{-5}$; 9) 20.9 and $7 \cdot 10^{-5}$; 10) 35.5 and $7 \cdot 10^{-5}$; 11) 10.2 and $6.6 \cdot 10^{-5}$; 12) 10.2 and $5 \cdot 10^{-4}$; 13) 10.2 and $3.3 \cdot 10^{-5}$; 14) 10.2 and $5 \cdot 10^{-4}$. 1)-8) For polyox 301, authors' data; 9)-10) for polyacrylamine, authors' data; 11)-14) respectively polyox 301, P35, PC1, 205, Fabula's data; I) $\lambda = 64/Re$; II) $1/\sqrt{\lambda} = 9.6 \lg Re\sqrt{\lambda} - 19.2$; III) $1/\sqrt{\lambda} = 9.2 \lg Re\sqrt{\lambda} - 19.6$; IV) $1/\sqrt{\lambda} = 2 \lg Re\sqrt{\lambda} - 0.8$.

For the limiting form of the velocity profile of polymer solutions flowing in a tube, [Eq. (1)] Eq. (7) gives the following relationship between the velocity values in question:

$$v_s = U - 18.5 v_*. \tag{9}$$

In order to determine the resistance law for the general case of polymer solutions flowing in a tube we may use Eqs. (3) and (4), which are satisfied on the tube axis:

$$U/v_* = 5.75 \lg (Rv_*/\nu_{sol}) + 20.25 \lg (y_1 v_*/\nu_{sol}) - 18.2. \tag{10}$$

Allowing for the relationships $Re = v_s d/\nu_{sol}$ and $v_* = v_s\sqrt{\lambda/8}$, this condition enables us to express the resistance law in the following form:

$$1/\sqrt{\lambda} = 9.2 \lg Re \sqrt{\lambda} + C - 14.7, \tag{11}$$

where

$$C = 7.16 \lg \gamma + 1.7\gamma^2 - 6.8\gamma. \tag{12}$$

Corresponding to the form of the velocity profile of polymer solutions defined by Eq. (1), we have the resistance law

$$1/\sqrt{\lambda} = 9.2 \lg Re \sqrt{\lambda} - 19.8, \tag{13}$$

derived from the general law (11) for the condition $\gamma = 1$. In the range of Reynolds numbers $Re = 2 \cdot 10^4$–$5 \cdot 10^5$ the maximum reduction in the flow resistance of polymer solutions defined by Eq. (13) lies in the range 75-80%. This reduction in resistance over the Reynolds number range indicated is supported by the many experimental results [8, 4] presented in Fig. 4. By extrapolating the universal relationship of Eq. (13) we may convince ourselves that the reduction which takes place in the frictional resistance during the flow of a polymer solution of limiting concentration in a tube (when the whole flow region up to $y_1 = R$ constitutes an expanded "buffer" zone) is subject to a scale effect, which increases the extent of the reduction in resistance on passing to large Reynolds numbers.

Preliminary calculations of the relationships $F = f(\gamma)$ and $C = f_1(\gamma)$ by means of Eq. (8) and (12) are presented graphically in Fig. 3; these enable us to predict the form of the velocity profile for specified flow parameters (resistance coefficient λ and Reynolds number Re). According to Eq. (11), the value of the free term in this equation is given by

$$C = 1/\sqrt{\lambda} - 9.2 \lg \mathrm{Re} \sqrt{\lambda} + 14.7. \tag{14}$$

For the value of C calculated by means of Eq. (14) we may determine $\gamma = y_1/R$ from Fig. 3.

The form of the velocity profile for specified flow conditions expressed in semilogarithmic coordinates $u/v_* - \lg(yv_*/\nu_{sol})$ will be determined by the law of Eq. (1) up to a value of the dimensionless coordinate $(y_1 v_*/\nu_{sol})$, and by that of Eq. (3) from this value to the tube axis. For example, the velocity profile representing the flow of a solution of guar resin of concentration $c = 1.5 \cdot 10^{-4}$ g/cm^3 was determined for the condition $\lambda = 0.00845$ and $\mathrm{Re} = 1.49 \cdot 10^5$. Under these conditions $\gamma = y_1/R = 0.016$. Remembering that $v_* = 0.196$ m/sec, $\nu_{sol} = 1.4 \cdot 10^{-6}$ m^2/sec, we may calculate the dimensionless value of the coordinate y_1

$$\lg \frac{y_1 v_*}{\nu_{sol}} = 1.62,$$

which agrees closely with measurements of the velocity profile made under the same conditions (Fig. 2, points 6).

In determining the maximum velocity on the tube axis for flows of polymer solutions with concentrations equal to the limiting value or over, the concept of the three-layer model of the velocity profile in the tube is clearly insufficient. In this case it is difficult to choose a value meaningful of the parameter for a value of $C \cong 5$ since values of $\gamma = 0.5$-1 may be made to fit the latter. However, the change in the function F over this range is very considerable. In this case we shall have to allow for the existence of an outer (fourth) zone of the velocity profile; an estimation of the extent of this zone carried out for water flows indicated that is reached from approximately $y/R = 0.5$ to the tube axis. On the logarithmic scale $(\lg(yv_*/\nu_{sol}))$ this zone occupies a very small part of the profile.

The extent of the outer zone corresponding to the flows of polymer solutions of limiting concentration may be considered as having the same dimensions as those deduced from measurements of the velocity profile of a flow of polyox solution with a concentration of $c = 2.6 \cdot 10^{-5}$ g/cm^3 (Fig. 2). The maximum velocity in this case may be defined thus:

$$U = v_S + Fv_*,$$

where $F = +12.6$ (for $y_1/R = 0.5$).

According to measurements of this profile $v_S = 1.75$ m/sec; $v_* = 0.0419$ m/sec and $U = 1.75$ m/sec $+ 12.6 \cdot 0.0419$ m/sec $= 2.28$ m/sec, which exactly corresponds to the experimentally-measured value.

Equations (6)-(8), which describe the velocity profile for the flow of a polymer solution in a tube, may be extended to the case of the flow of a polymer solution of constant concentration around a plate. Here the frictional resistance coefficient of the plate for the greatest feasible reduction in resistance will be given by

$$c_f = \frac{0.78}{\mathrm{Re}^{0.275}}, \quad \mathrm{Re} < 10^8. \tag{15}$$

Relationships analogous to our own were derived by Virk [9] for the form of velocity profile associated with the flow of a polymer solution in a tube. The only difference lies in slight discrepancies between the constants encountered in Eqs. (1) and (13) (Fig. 4).

Our investigations into the velocity profiles of turbulent flows of polyox, guar resin, and polyacrylamide in a tube have thus confirmed the general character of these flows.

The reduction in resistance increases with increasing mean velocity, and especially with increasing concentration of the solution; at the same time the buffer zone increases in size by virtue of a reduction in the extent of the developed turbulent region. Corresponding to the maximum reduction in resistance, we find an increase in the buffer zone, extending right to the tube axis. The concentration of the solution corresponding to this limiting case should be regarded as optimum, since any further increase in concentration will not alter the character of the flow, and hence the reduction in resistance will be solely determined by the Reynolds number, as indicated in Eq. (15).

Equations (8) and (13) and the graphical relationships $F(\gamma)$ and $C(\gamma)$ of Fig. 3 enable us to determine the flow resistance of a solution in a tube from the shape of the velocity profile, or, on the other hand, to predict the shape of the profile from the resistance and Reynolds number.

NOTATION

d	is the tube diameter;
r	is the distance from tube axis;
R	is the tube radius;
y	is the distance from tube wall;
v_S	is the mean velocity (flow rate) of the liquid in the working tube;
ρ	is the density of liquid;
ν	is the kinematic viscosity of water;
ν_{sol}	is the kinematic viscosity of the solution;
c	is the weight concentration;
Re	is the Reynolds number;
τ_W	is the tangential stress of wall friction for the flow of water;
τ_{sol}	is the tangential stress of wall friction for the flow of polymer solution;
$\lambda = 8\tau_{sol}/\rho\gamma_S^2$	is the resistance coefficient;
u	is the velocity in the tube cross section;
U	is the maximum velocity on the tube axis;
$v_* = \sqrt{\tau_{sol}/\rho}$	is the dynamic velocity;
y_1	is the thickness of buffer sublayer;
$S = (\tau_W - \tau_{sol}/\tau_W)$	is the reduction in resistance.

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